Slides on particle dispersal, pressure equilibration, and entropy

CH102 Spring 2016, A1 and A2 lecture 27

- Enumerating particle dispersal
- Practice with particle dispersal
- Maximum particle dispersal = uniform pressure
- Arrangements → entropy

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Blind chance & dumb luck

Everything—*absolutely everything*—that happens, happens *solely* because of *blind chance* and *dumb luck*.

To quantify this …

1. Learn to *count* the ways
2. Search for *greatest number* of ways

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Counting *distinguishable* (unique) arrangements

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Counting *distinguishable* (unique) arrange

Say we have three girls and four boys.

What is the probability of calling them into line in the order $gggbbbb$?

$$\frac{3}{7} \times \frac{2}{6} \times \frac{1}{5} = \frac{1}{35}$$

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Counting *distinguishable* (unique) arrange

Say we have three girls and four boys.

What is the probability of calling them into line in the order $bgbgbgb$?

$$\frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{35}$$

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Counting *distinguishable* (unique) arrange

Say we have three girls and four boys.

If we ignore whether a child is boy or girl, what is the total number of arrangements?

$$7 \times 6 \times 5 \ldots \times 1 = 7! = 5040$$

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Counting *distinguishable* (unique) arrange

Say we have three girls and four boys.

For each particular arrangement, say $bgbgbgb$, how many ways can it come about?

$$3!\,4! = 144$$

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Counting *distinguishable* (unique) arrange

Say we have three girls and four boys.

This means the total number of arrangements can be expressed as

$$7! = W \times 3!\,4!$$

$$5040 = 144\,W$$

$$W = 5040/144 = 35$$

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Counting *distinguishable* (unique) arrange

More generally, say we have j girls and k boys.

The number of unique arrangements of n_1 objects of one kind and n_2 object of another kind is

$$W(j,k) = \frac{j+k}{j!\,k!}$$

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Practice with particle dispersal

See page 3 of
http://quantum.bu.edu/courses/ch102-spring-2016/notes/SecondLaw.pdf

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[TP] How many distinguishable ways can 3 water molecules and 2 ink molecules be arranged?

0% 1. 8
0% 2. 10
0% 3. 12
0% 4. 120
0% 5. None of these

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Response Counter

10

20

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[TP] How many distinguishable ways can 5 water molecules and 2 ink molecules be arranged?

0% 1. 14
0% 2. 21
0% 3. 240
0% 4. 5040
0% 5. None of these

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Response Counter

10

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[TP] How many distinguishable ways can 2 ink molecules be arranged among 12 water molecules?

0% 1. 36
0% 2. 240
0% 3. 455
0% 4. 720
0% 5. None of these

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Response Counter

10

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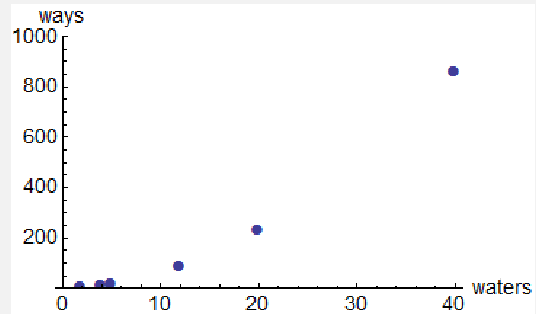
The more water, the more ways ink disperses

2 ink particles in water



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Maximum particle dispersal = uniform pressure

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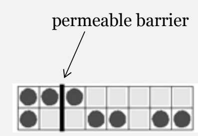
Pressure in a gas is unequal



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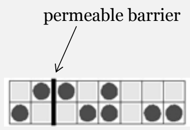
Pressure in a gas is uniform



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Pressure in a gas becomes uniform

Why?

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Lattice gas model of pressure

$1/(R\ T)\ P = n/V$ = gas density

n = particles

V = lattice positions

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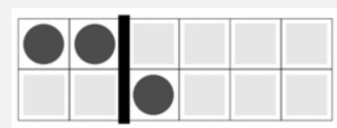
$P_{\text{left}} > P_{\text{right}}$



Left side: $n/V = 2/4$, $W_{\text{left}} = \ldots$

6

Right side: $n/V = 1/8$, $W_{\text{right}} = \ldots$

8

$W_{\text{total}} = W_{\text{left}} \times W_{\text{right}} = 6 \times 8 = 48$

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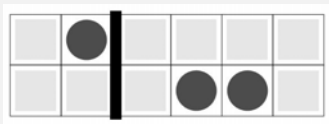
$P_{left} = P_{right}$



Left side: $n/V = 1/4$, $W_{left} = \ldots$

4

Right side: $n/V = 2/8$, $W_{right} = \ldots$

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$W_{total} = W_{left} \times W_{right} = 4 \times 28 = 112$

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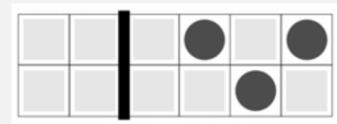
$P_{left} < P_{right}$



Left side: $n/V = 0/4$, $W_{left} \ldots$

1

Right side: $n/V = 3/8$, $W_{right} = \ldots$

56

$W_{total} = W_{left} \times W_{right} = 1 \text{ x } 56 = 56$

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Pressure in a gas becomes uniform

Why?

$P_{left} > P_{right}$ has $W_{total} = 48$

$P_{left} = P_{right}$ has $W_{total} = 112$

$P_{left} < P_{right}$ has $W_{total} = 56$

Uniform pressure *maximizes W*!

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Arrangements → Entropy

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$S = k_B \ln(W)$

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$S = k_B \ln(W)$

Why natural log?

Doubling size of system: $W \rightarrow W \times W = W^2$

Doubling size of system: $S \rightarrow 2\,S$, so …

Boltzmann's definition makes S *scale with size of system* (extensive).

$k_B = R/N_A = 1.4 \times 10^{-23}$ J/K

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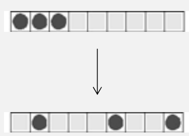
Spontaneous?



Calculate the entropy change.

$W_i = 1 \rightarrow W_f = (6 + 3)!/(6!\ 3!) = 84$

$\Delta S = S_f - S_i = k_B \ln(W_f/W_i) = k_B \ln(84/1) > 0$

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