

Titration-region & buffer calculations

CH102 Spring 2011
Boston University



An acid-base region is **a circumstance**

... **not** a sequence of operations

Region 1: **Only weak** acid (base)

Region 2: **Both weak** acid (base) and its **conjugate** base (acid)

Region 3: **Only conjugate** of weak acid (base)

Region 4: **Strong** base (acid), no matter what else

For each of these circumstances ...

it **does not matter how they arise**



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Region 1: Strong acid only

$$\text{HA}(aq) + \text{H}_2\text{O}(l) \rightarrow \text{H}_3\text{O}^+(aq) + \text{A}^-(aq)$$

	HA	H ₃ O ⁺	A ⁻
Initial	c_a	10^{-7}	0
Revised initial, $K_a > 1$	0	$10^{-7} + c_a \approx c_a$	c_a
Change	+ x	- x	- x
Equilibrium	x	$c_a - x \approx c_a$	$c_a - x \approx c_a$

$10^{-7} + c_a \approx c_a$ because $K_a > K_w$
 $c_a - x \approx c_a$ because $K_a > 1$
 $(\text{HA}) = x \approx c_a^2/K_a$

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Region 1: Strong acid only example


$$\text{HA}(aq) + \text{H}_2\text{O}(l) \rightarrow \text{H}_3\text{O}^+(aq) + \text{A}^-(aq)$$

K_a of HA is 1×10^6

What is the pH and (HA) in 0.001 M HA?

$(\text{H}_3\text{O}^+) = c_a = 0.001 \rightarrow \text{pH} = 3.0$

$(\text{HA}) = x = c_a^2/K_a = (0.001)^2/(1 \times 10^6) = 1 \times 10^{-12} !!!$

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Region 1: Weak acid only

$$\text{HA}(aq) + \text{H}_2\text{O}(l) \rightarrow \text{H}_3\text{O}^+(aq) + \text{A}^-(aq)$$

	HA	H ₃ O ⁺	A ⁻
Initial	c _a	10 ⁻⁷	0
Change	-x	+x	+x
Equilibrium	c _a - x ≈ c _a	10 ⁻⁷ + x ≈ x	x

10⁻⁷ + x ≈ x because K_a > K_w
 c_a - x ≈ c_a because 1 > K_a
 (H₃O⁺) = x ≈ √(K_a c_a)

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Region 1: Weak acid only example


$$\text{HA}(aq) + \text{H}_2\text{O}(l) \rightarrow \text{H}_3\text{O}^+(aq) + \text{A}^-(aq)$$

K_a of HA is 1 × 10⁻⁷

What is the pH of 0.001 M HA?

$$(\text{H}_3\text{O}^+) = \sqrt{K_a c_a} = \sqrt{1 \times 10^{-10}} = 1 \times 10^{-5}$$

pH = 5.0

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Region 2: Partially neutralized

$$\text{HA}(aq) + \text{H}_2\text{O}(l) \rightarrow \text{H}_3\text{O}^+(aq) + \text{A}^-(aq)$$

	HA	H ₃ O ⁺	A ⁻
Initial	c_a	10^{-7}	c_b
Change	$-x$	$+x$	$+x$
Equilibrium	$c_a - x \approx c_a$	$10^{-7} + x \approx x$	$c_b + x \approx c_b$

$10^{-7} + x \approx x$ because $K_a > K_w$
 $c_a - x \approx c_a$ and $c_b + x \approx c_b$ because $1 > K_a$
 $(\text{H}_3\text{O}^+) = x \approx K_a (c_a/c_b) = K_a (\text{mol}_a/\text{mol}_b)$

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Region 2: Partially neutralized example


$$\text{HA}(aq) + \text{H}_2\text{O}(l) \rightarrow \text{H}_3\text{O}^+(aq) + \text{A}^-(aq)$$

K_a of HA is 1×10^{-5}

1.0 L each of 0.012 M HA and 0.010 M NaOH are combined. What is the pH?

$\text{mol}_a = 0.002 \text{ mol}$, $\text{mol}_b = 0.010 \text{ mol}$
 $(\text{H}_3\text{O}^+) = x = K_a (\text{mol}_a/\text{mol}_b) = 2 \times 10^{-6}$

$\text{pH} = 6 - 0.3 = 5.7$

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Region 3: Equivalence point

$$\text{H}_2\text{O}(l) + \text{A}^-(aq) \rightarrow \text{HA}(aq) + \text{OH}^-(aq)$$

	A ⁻	HA	OH ⁻
Initial	c_b	0	10^{-7}
Change	$-x$	$+x$	$+x$
Equilibrium	$c_b - x \approx c_b$	x	$10^{-7} + x \approx x$

$10^{-7} + x \approx x$ because $K_b = K_w/K_a \gg K_w$
 $c_b - x \approx c_b$ because $1 \gg K_b$
 $(\text{OH}^-) = x \approx \sqrt{K_b c_b}$

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Region 3: Equivalence point example

$$\text{H}_2\text{O}(l) + \text{A}^-(aq) \rightarrow \text{HA}(aq) + \text{OH}^-(aq)$$

K_a of HA is 1×10^{-7}


1.0 L each of 0.002 M HA and 0.002 M NaOH are combined. What is the pH?

$$c_b = 0.002 \text{ mol}/(2 \text{ L}) = 0.001 \text{ M}$$

$$K_b = K_w/K_a = 1 \times 10^{-7}$$

$$(\text{OH}^-) = \sqrt{K_b c_b} = \sqrt{(1 \times 10^{-7})(0.001)} = 1 \times 10^{-5}$$

$$\text{pOH} = 5.0 \rightarrow \text{pH} = 9.0$$

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Region 4: Beyond equivalence example


$$\text{H}_2\text{O}(l) + \text{A}^-(aq) \rightarrow \text{HA}(aq) + \text{OH}^-(aq)$$

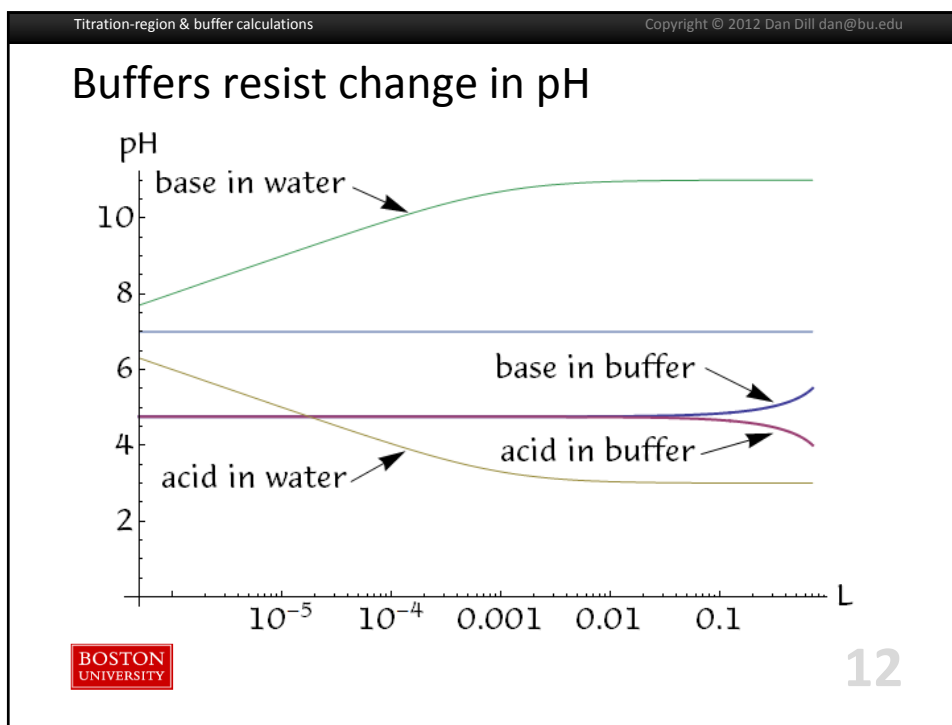
K_a of HA is 1×10^{-7}

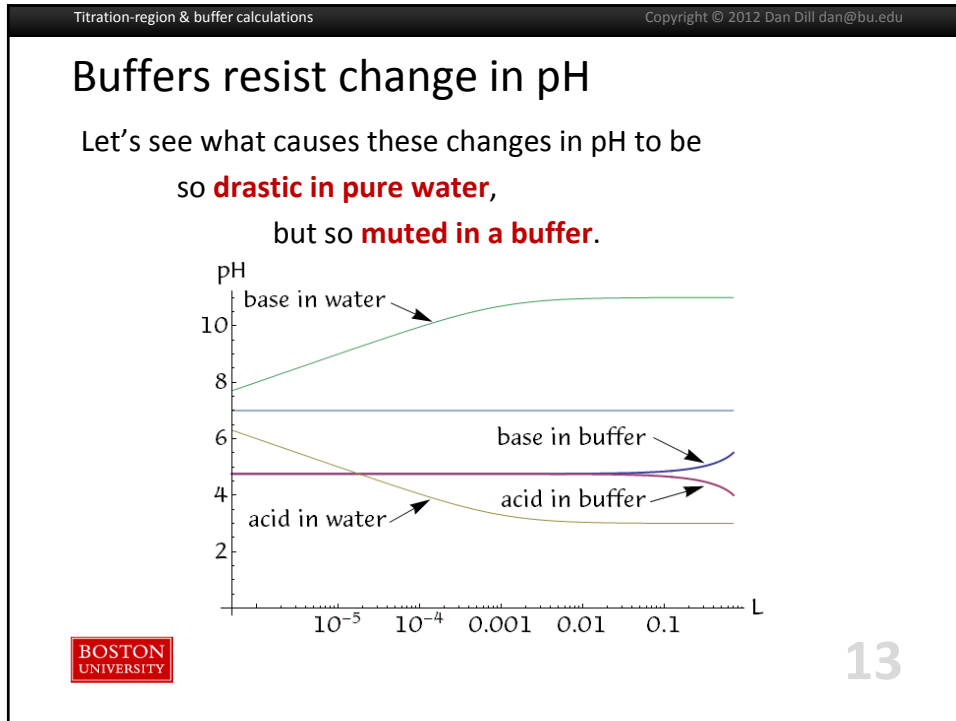
1.0 L each of 0.002 M HA and 0.003 M NaOH are combined. What is the pH?

$$[\text{OH}^-] = 0.001 \text{ mol}/(2 \text{ L}) = 0.0005 \text{ M}$$

$$\text{pOH} = 4 - 0.7 = 3.3 \rightarrow \text{pH} = 10.7$$

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Buffers resist change in pH

Added strong base (say, OH⁻) is gobbled up ...

$$\downarrow \text{HA}(aq) + \text{OH}^-(aq) \rightarrow \text{H}_2\text{O}(l) + \uparrow \text{A}^-(aq)$$

c_a lowered, c_b raised, c_a/c_b lowered

Added strong acid (say, HCl) is gobbled up ...

$$\text{HCl}(aq) + \downarrow \text{A}^-(aq) \rightarrow \uparrow \text{HA}(aq) + \text{Cl}^-(aq)$$

c_b lowered, c_a raised, c_a/c_b raised

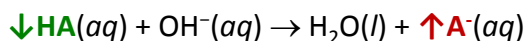
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Add **strong base** to buffer

1 L buffer, $c_a = c_b = 1.00 \text{ M}$, $K_a = 1 \times 10^{-5}$, **pH = ...**

5.00

Add 100. mL of 0.100 M NaOH



HA $\rightarrow 1.00 \text{ mol} - 0.010 \text{ mol} = \mathbf{0.99 \text{ mol}}$

A⁻ $\rightarrow 1.00 \text{ mol} + 0.100 \text{ mol} = \mathbf{1.01 \text{ mol}}$

$c_a = 0.99 \text{ mol}/1.10 \text{ L}$, $c_b = 1.01 \text{ mol}/1.10 \text{ L}$

$c_a/c_b = 1.00 \rightarrow \mathbf{0.99/1.01}$, **pH $\rightarrow 5.01$ (tiny change!)**

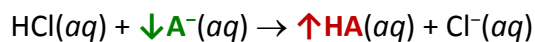


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Add **strong acid** to buffer

1 L buffer, $c_a = c_b = 1.00 \text{ M}$, $K_a = 1 \times 10^{-5}$, **pH = 5.00**

Add 100. mL of 0.100 M HCl



HA $\rightarrow 1.00 \text{ mol} + 0.010 \text{ mol} = \mathbf{1.01 \text{ mol}}$

A⁻ $\rightarrow 1.00 \text{ mol} - 0.010 \text{ mol} = \mathbf{0.99 \text{ mol}}$

$c_a = 1.01 \text{ mol}/1.10 \text{ L}$, $c_b = 0.99 \text{ mol}/1.10 \text{ L}$

$c_a/c_b = 1.00 \rightarrow \mathbf{1.01/0.99}$, **pH $\rightarrow 4.99$ (tiny change!)**



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Add strong acid/base to water

1 L of water, $K_a = 1 \times 10^{-14}$, **pH = 7.00**

Add 100. mL of 0.100 M HCl
 $(\text{H}_3\text{O}^+) = 0.010 \text{ mol}/1.10 \text{ L} = 0.0091$
pH = 2.04 (huge change!)

Add 100. mL of 0.100 M NaOH
 $(\text{OH}^-) = 0.010 \text{ mol}/1.10 \text{ L} = 0.0091$
 pOH = 2.04, **pH = 11.96 (huge change!)**

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