CH 102 Take home Discussion Quiz 10 (Attention: Quiz is 2 pages double sided)

Name ________________________________ Section/ Day___________________

(All work must be shown to receive credit.)

Calculating arrangements due to distribution of molecules: $W_{pos}(a,b,c,\ldots) = \frac{(a+b+c+\ldots)!}{a!b!c!\ldots}$

Where a, b, c, ...— number of different particles

Calculating arrangements due to distribution of quanta (q) of energy among particles (m) or with (p) partitions:

$$W(m,q) = \frac{(q+p)!}{q!\,p!} = \frac{(q+(m-1))!}{q!(m-1)!} \quad \text{where } p=m-1$$

1. (***1point***) Calculate the number of distinguishable arrangements of 3 objects A and 2 objects B in 8 boxes.

$$W = \frac{(3+2+3)!}{3!2!3!} = 560$$

2. (2points) The figure below describes an osmosis process consisting of water (W) and methanol (M) molecules.

			W	W	W	
	W		W	W	M	
W		W	W	W	M	

a) Calculate the number of distinguishable configurations for the system above ($W_{system} = W_{left} * W_{right}$).

$W_{system} = W(3,6)*W(2,7,3) = \frac{(3+6)!}{3!6!}\cdot\frac{(2+7+3)!}{2!7!3!} = \frac{12!}{2!7!3!}\cdot\frac{9!}{3!6!} = 665{,}280$

Does the arrangement above have the maximum number of configurations? Prove your answer.

No it does not.

(1 water moved to the left) $W_{system} = W(4,5)*W(2,6,4) = \frac{(4+5)!}{4!5!}\cdot\frac{(2+6+4)!}{2!6!4!} = \frac{12!}{2!6!4!}\cdot\frac{9!}{4!5!} = 1{,}746{,}360$

(2 water moved to the left) $\mathbf{W_{system} = W(5,4)*W(2,5,5)} = \frac{(5+4)!}{5!4!}\frac{(2+5+5)!}{2!5!5!} = \mathbf{2{,}095{,}632}$

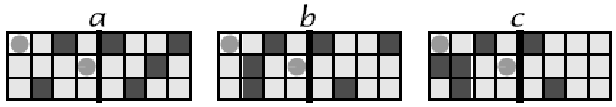
(3 water moved to the left) $W_{system} = W(6,3)*W(2,4,3) = \frac{(6+3)!}{3!6!}\cdot\frac{(2+4+6)!}{2!4!6!} = 1{,}164{,}240$

(1 water moved to the right) $W_{system} = W(2,7)*W(2,8,2) = \frac{(2+7)!}{3!6!}\cdot\frac{(2+8+3)!}{2!8!3!} = 106{,}920$

b) The thick black line represents a semipermeable (permeable only to water) membrane. If the number of particles can pass through the membrane do so until there are an equal number on both sides, what will the new number of arrangements (W_{system}) be?

(2 water moved to the left) $W_{system} = W(5,4)*W(2,5,5) = \frac{(5+4)!}{5!4!}\frac{(2+5+5)!}{2!5!5!} = 2{,}095{,}632$

3. (2 points) The diagrams below show three different distributions of two particles of one kind (light circles) and six particles of another kind (dark squares). The membrane (vertical heavy line) is impermeable to all particles.



Find the number of distinguishable arrangements in box a. (Hint: $W_{total} = W_{left\ side} \cdot W_{right\ side}$)

$$W_{total}= \frac{(2+2+8)!}{2!2!8!} * \frac{(4+8)!}{4!8!} = 1{,}470{,}150$$

a. Find the number of distinguishable arrangements in box b.

$$W_{total}= \frac{(2+3+7)!}{2!3!7!} * \frac{(3+9)!}{3!9!} = 1{,}742{,}400$$

Which box has the maximum number of arrangements? What do you think is the reason for this box to have maximum number of arrangements? **Box b** has the same number of dark squares on both sides.

b. Of the choices below, which transformations are spontaneous? (Hint: Do you really need to do the calculations to answer this question?) ***a→b***

4. (2 points) Consider the system of two identical four-atom solids that are in contact with each other, one having four quanta of energy, and the other having eight quanta.

Calculate W_{system} for the system described:

$$\boldsymbol{W_{sysA}} = W(4,4q) \cdot W(4,8q) = \frac{(4+4-1)!}{4!(4-1)!} \cdot \frac{(8+4-1)!}{8!(4-1)!} = 35*165=5775$$

$$W(4,4q) = \frac{(4+4-1)!}{4!(4-1)!} = \frac{7!}{4!3!} = \frac{5 \cdot 6 \cdot 7}{1 \cdot 2 \cdot 3} = 35$$

$$W(4,8q) = \frac{(8+4-1)!}{8!(4-1)!} = \frac{11!}{8!3!} = \frac{9 \cdot 10 \cdot 11}{1 \cdot 2 \cdot 3} = 165$$

a. Calculate the new W_{system} if two quanta of energy are transferred from the solid with four quanta to the solid with eight quanta.

$$\boldsymbol{W_{sysB}} = W(4,2q) \cdot W(4,10q) = \frac{(2+4-1)!}{2!(4-1)!} \cdot \frac{(10+4-1)!}{10!(4-1)!} = 10*286=2860$$

$$W(4,2q) = \frac{(2+4-1)!}{2!(4-1)!} = \frac{5!}{2!3!} = \frac{4 \cdot 5}{1 \cdot 2} = 10 ; \quad W(4,10q) = \frac{(10+4-1)!}{10!(4-1)!} = \frac{13!}{10!3!} = \frac{11 \cdot 12 \cdot 13}{1 \cdot 2 \cdot 3} = 286$$

b. Would the process of transferring of quanta of energy described in **part b** correspond to a spontaneous event or non-spontaneous event? Please give a brief explanation as to why or why not.

$\boldsymbol{W_{sysA}} = W_{in} = 5775$ $\boldsymbol{W_{sysB}} = W_{final} = 2860$ $W_f / W_{in} = \frac{2860}{5775} < 1 \rightarrow \Delta S < 0$ not spontaneous.

Process is not spontaneous because number of distinguishable arrangements decreases

5. (2points) Consider 3 "X" particles and 2 quanta of energy in 4 boxes (shown below). If one of the particles and one quantum of energy removed, calculate the change in entropy in J/K to one significant figure.

X	X
	X

$$W_{sys(initial)}=W_p \cdot W_q=\frac{(3+1)!}{3!1!}\cdot\frac{(2+3-1)!}{2!(3-1)!}=4\cdot 6=24$$

$$W_{sys(final))}=W_p \cdot W_q=\frac{(2+2)!}{2!2!}\cdot\frac{(1+2-1)!}{1!(2-1)!}=6\cdot 2=12$$

$$\mathbf{\Delta S_{system}}= k_b\, 2.3\log\frac{W_{final}}{W_{initial}}=1.38\ 10^{-23}\ \text{J/K}\cdot 2.3\cdot \log 0.5= -1*10^{-23}\ \text{J/K}$$

Not spontaneous

6. (1point) In one process : a → b ($\boldsymbol{W_b = 100 + W_a}$)
In another process: c → d ($\boldsymbol{W_d = 1000 + W_c}$)
Which process has the greater entropy change (ΔS)**?** Keep in mind that ***W***a, ***W***b, ***W***c and ***W***d represent the number of distinguishable arrangements of a, b, c and d, respectively.
Circle the correct answer:

a → b c → d **More information needed**

Prove your answer:

ΔS (a → b) ~ ln W_b / Wa = ln ((100 + W_a)/ W_a)

ΔS (c → d) ~ ln W_d / Wc = ln ((1000 + W_c)/ W_d)

In case of compare ΔS (a → b) and ΔS (c → d) we need to compare which fraction is lager W_b / Wa or W_d / Wc

W_b / Wa = (100 + W_a)/ W_a= 100/ W_a +1

W_d / Wc = (1000 + W_c)/ W_d = 1000/W_d +1