Lecture 5 CH131 Summer 1

Wednesday, May 29, 2019

The will be lab today Wednesday, May 29

- Complete: Ideal gas law
- Partial pressures
- Kinetic theory of gases
- Practice with particle picture of gases

Next lecture: Continue 9.1–9.6: Calculation of molecular speeds; Distribution of speeds; How intermolecular attraction affects gas behavior; How molecular size affects gas behavior; Gas law for real gases: van der Waals equation

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[TP] The volume of a balloon filled with 1300 mol $H_2(g)$ at 23 °C and 1 bar is …
[Recall that R = 8.314 J /(K mol)]

0%	1.	2500 L
0%	2.	250,000 L
65%	3.	32,000 L
29%	4.	3,200,000 L
6%	5.	None of the above

$V = \frac{nRT}{P}$

$= \frac{1300\ \cancel{mol} * 8.3\ Pa\ m^3}{\cancel{K}\ \cancel{mol}} \frac{296\ \cancel{K}}{1\ bar\ (10^5\ Pa)} \frac{* 10^3\ L}{m^3}$

$= \frac{1.3\times10^3 * 8.3 * 296 * 10^3\ L}{10^5\ P}$

$\approx 1.3 * 10 * 8.3 * 296$

$Pa = \frac{J}{m^3}$

$J = Pa\ m^3$

$m^3 = 10^3\ L$

$T = 273.13 + 23$

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Using ideal gas law

22 L of He is stored at 152 bar and 31°C. How many balloons can each be filled with 5.0 L of He at 1.00 bar and 22 °C?

Answer: k = 650

$n = \frac{P_1V_1}{RT_1} = \frac{152\ bar * 22\ L}{R\ 304\ K}$

$x * 5\ L = V_{total} = \frac{nRT_2}{P_2}$

$= \frac{P_1V_1}{\cancel{R}T_1} \cdot \frac{\cancel{R}T_2}{P_2}$

$x = \frac{P_1}{P_2} \cdot \frac{T_2}{T_1} \cdot \frac{V_1}{5\ L} = \frac{152\ \cancel{bar}}{1.00\ \cancel{bar}} \cdot \frac{295\ \cancel{K}}{304\ \cancel{K}} * \frac{22\ \cancel{L}}{5\ \cancel{L}}$

≈ 650

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Mixtures of gases

Dalton's law of partial pressures

$2.0\ atm = y_{O_2}\ 5.0\ atm$ $\quad 5 = 2 + 3$

$y_{O_2} = \frac{2.0}{5.0}$

$P_{O_2} = \frac{n_{O_2} RT}{V}$

$P_{N_2} = \frac{n_{N_2} RT}{V}$

$n = n_{O_2} + n_{N_2}$

$P = \frac{nRT}{V} = (n_{O_2} + n_{N_2}) \frac{RT}{V}$

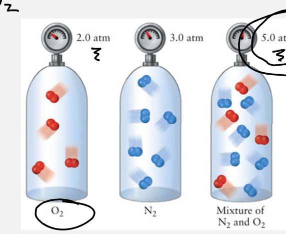
$= P_{O_2} + P_{N_2}$

$P_{O_2} = \frac{n_{O_2}}{n}\left(\frac{RT}{V} * n\right) = \frac{n_{O_2}}{n} P$

$\frac{n_{O_2}}{n_{O_2}+n_{N_2}} = x_{O_2}$

$\frac{n_{N_2}}{n_{O_2}+n_{N_2}} = x_{N_2}$

$x_{O_2} + x_{N_2} = 1$



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Mixtures of gases: Problem 9.33

33. Sulfur dioxide reacts with oxygen in the presence of platinum to give sulfur trioxide:

$$\underset{26.0}{2\,SO_2(g)} + \underset{83.0}{O_2(g)} \longrightarrow \underset{17.0}{2\,SO_3(g)}$$

Suppose that at one stage in the reaction, 26.0 mol SO_2, 83.0 mol O_2, and 17.0 mol SO_3 are present in the reaction vessel at a total pressure of 0.950 atm. Calculate the mole fraction of SO_3 and its partial pressure.

$$P_{SO_3} = \chi_{SO_3} P$$

$$\chi_{SO_3} = \frac{17.0}{26.0 + 83.0 + 17.0} = 0.135$$

$$P_{SO_3} = 0.135 \times 0.950\ \text{atm} = 0.128\ \text{atm}$$

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Kinetic-molecular theory of gases

$$P = \frac{nRT}{V}$$

Goal: Get microscopic expression for pressure P

Key idea 1: Pressure is due to force exerted by particles during collisions with the container walls

Key idea 2: Force is due to momentum change in collision with the container walls.

Note: Here upper-case P is used for pressure and lower-case p is used for momentum.

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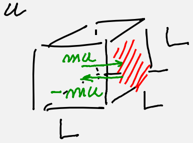
Kinetic-molecular theory of gases

$$V = L^3 \qquad A = L^2$$

$$P = \frac{F}{A} \qquad F = \frac{\Delta p}{\Delta t} \qquad p = mu$$



$$\Delta p_{part} = p_f - p_i = -mu - (mu) = -2mu$$

$$\Delta p_{wall} + \Delta p_{part} = 0 \quad \text{elastic.}$$

$$\Delta p_{wall} = -\Delta p_{part} = +2mu$$

$$F = \frac{\Delta p_{wall}}{\Delta t} = \frac{2mu}{\Delta t}$$

$$\Delta t = \frac{2L}{u}$$

$$F = \frac{\cancel{2}mu}{\cancel{2}L/u} = \frac{mu^2}{L}$$

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Kinetic-molecular theory of gases

$$F = \frac{\Delta p}{\Delta t} = \frac{2mu}{2L/u} = mu^2/L$$

$$P = \frac{F}{A} = \frac{F}{L^2} = \frac{mu^2}{L^3} = \frac{mu^2}{V}$$

$$PV = mu^2 \overset{?}{=} nRT$$

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Kinetic-molecular theory of gases

$P_1 = \frac{m u_1^2}{V}$

$P = P_1 + P_2 + \cdots + P_N$

$= \frac{Nm}{V}\left(\frac{u_1^2 + u_2^2 + u_3^2 + \cdots + u_N^2}{N}\right)$

u^2_{avg}

# of particles $= N$

$= n N_A$

total mass

$Nm = n N_A m = n M$

$= \frac{Nm\, u^2_{avg}}{V}$

$P_{1D} = \frac{n M u^2_{avg}}{V} \Rightarrow P_{3D} = \frac{n M u^2_{avg}}{3V}$

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Kinetic-molecular theory of gases

$P = \frac{n M u^2_{avg}}{3V} = \frac{nRT}{V}$

$u^2_{avg} = \frac{3RT}{M}$

$= \frac{nM}{3V}\,\frac{3RT}{M}$

heavies slower

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Kinetic-molecular theory of gases

Force due to j^{th} particle of mass m and speed u_j is $\Delta p/\Delta t$ …

$\Delta p = 2mu_j$ (elastic collision)

$\Delta t = 2L/u_j$ (travel time to opposite wall and back)

$F = \Delta p/\Delta t = mu_j^2/L$

Pressure due to j^{th} particle of mass m and speed u_j …

$$P_j = \frac{F}{\text{area}} = \frac{F}{L^2} = \frac{mu_j^2}{L^3} = \frac{mu_j^2}{V}$$

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Kinetic-molecular theory of gases

A single particle j exerts a pressure

$P_j = \frac{mu_j^2}{V}$

The total pressure P due to all of the N particles in the container is

$P = \frac{m}{V}(u_1^2 + u_2^2 + \cdots + u_j^2 + \cdots + u_N^2)$

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[TP] The different speeds, u_1, u_2, etc., in a gas is due to …

X 0% 1. collisions of gas particles with the walls of the container.
19% 2. collisions of gas particles with one another
0% 3. attractions between the particles of the gas and the particles of the walls of the container.
0% 4. attractions between the particles of the gas.
44% 5. 1 and 2
0% 6. 1 and 3
0% 7. 1, 2 and 3
38% 8. 1, 2, 3 and 4

Distribution of molecular speeds

root-mean-square

$u_{rms} = \sqrt{u^2_{avg}} = \sqrt{\frac{3RT}{M}}$

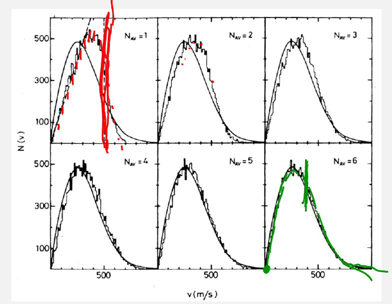
Here is what happens to the speeds of 20,000 particles, all initially at the same speed, after they each have undergone successive numbers of collisions.

Bonomo & Riggi,
Am. J. Phys. ,Vol 52, p 54 (1984)

Maxwell-Boltzmann distribution of speeds

$u^2_{avg} = \frac{3RT}{M}$

$\sqrt{u^2_{avg}} = \sqrt{}$



Kinetic-molecular theory of gases

We can rewrite the total pressure due to the N particles,

$$P = \frac{m}{V}(u_1^2 + u_2^2 + \cdots + u_j^2 + \cdots + u_N^2)$$

in terms of the average squared speed

$$u_{avg}^2 = (u_1^2 + u_2^2 + \cdots + u_j^2 + \cdots + u_N^2)/N$$

by multiplying and dividing P by N,

$$P = \frac{m}{V}N(u_1^2 + u_2^2 + \cdots + u_j^2 + \cdots + u_N^2)/N$$

$$= \frac{m}{V}Nu_{avg}^2 = \frac{m}{V}N_A n u_{avg}^2 = \frac{M}{V}nu_{avg}^2$$

Kinetic-molecular theory of gases

The expression for pressure,

$$P = \frac{nM}{V}u_{avg}^2 \text{ (one dimension)}$$

assumes the particle moves in just one dimension, back and forth between opposite walls, say the walls perpendicular to the x axis.

A more detailed treatment that takes into account motion in all three dimensions shows that the pressure on any one wall is only ⅓ as great,

$$P = \frac{nM}{3V}u_{avg}^2 \text{ (three dimensions)}$$

Calculation of molecular speeds

We now have two expressions for pressure:

The microscopic expression $P = \frac{nMu_{\text{avg}}^2}{3}/V$

and the macroscopic expression $P = nRT/V$

Comparing these, we get that $\frac{Mu_{\text{avg}}^2}{3} = RT$ ...

and so that the average squared speed is $u_{\text{avg}}^2 = \frac{3RT}{M}$

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Calculation of molecular speeds

$u_{\text{avg}}^2 = \frac{3RT}{M}$

is the connection between…

microscopic motion, quantified as u_{avg}^2, and …

the macroscopic concept temperature, T, and molar mass, M.

The square root of the mean (average) squared speed is the rms speed …

$u_{\text{rms}} = \sqrt{u_{\text{avg}}^2} = \sqrt{\frac{3RT}{M}}$

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Practice with particle picture of gases

Let's consider some questions to develop a particle-level understanding of why gases behave the way they do.

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[TP] Gas pressure is due to …

94% 1. collisions of gas particles with the walls of the container.
6% 2. collisions of gas particles with one another
0% 3. attractions between the particles of the gas and the particles of the walls of the container.
0% 4. attractions between the particles of the gas.
0% 5. 1 and 2
0% 6. 1 and 3
0% 7. 1, 2 and 3
0% 8. 1, 2, 3 and 4

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[Quiz] A container of volume V is filled with a gas at 20 °C. If V is decreased (while keeping T constant), the pressure P exerted by the gas on the walls of the container goes up ($P = nRT/V$). Why?

6%	1.	The particles move faster
0%	2.	The particles move slower
0%	3.	The particles hit the walls harder
0%	4.	The particles hit the walls less hard
94%	5.	The particles hit the walls more often
0%	6.	The particles hit the walls less often

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[TP] When more particles are added to the same V at the same T, P goes up ($P = nRT/V$). Why?

0%	1.	The particles move faster
0%	2.	The particles move slower
0%	3.	The particles hit the walls harder
0%	4.	The particles hit the walls less harder
100%	5.	More particles hit the walls in a given time
0%	6.	Fewer particles hit the wall in a given time

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[TP] When a gas is heated, if the P is to remain constant, then volume V must go up ($P = nRT/V$). Why?

0%	1.	The particles move faster
0%	2.	The particles move slower
0%	3.	The particles hit the walls harder
0%	4.	The particles hit the walls less harder
100%	5.	The distance travelled between collisions must increase
0%	6.	The distance travelled between collisions must decrease

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[TP] Two 1 L containers, A and B, each contain equal numbers of particles at 20 °C. The particles of gas in A are twice as heavy as those in B. What are the relative pressures in the two containers?

0%	1.	Pressure of A is half the pressure of B
100%	2.	Pressure of A equals the pressure of B
0%	3.	Pressure of A is twice the pressure of B
0%	4.	Pressure of A is four times the pressure of B

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Practice with particle picture of gases

Two 1 L containers, A and B, each contain equal numbers of particles at 20 °C. The particles of gas in A are twice as heavy as those in B. The pressure in the two containers is the same.

How can this be?

$$P = (nM/V)⅓\, u_{avg}^2 = (n\cancel{M}/V)⅓\,(3RT/\cancel{M})$$

Pressure depends on speed AND mass

Heavy particles move s l o w l y ! ! !

Light particles move q u I c k l y !!!

But at the same temperature, they exert the same pressure.

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[TP] Equal amounts of gases A and B are in a single container. The mass of a molecule of gas in A is twice that of the gas in B, $m_A = 2\, m_B$. The container is pierced with a hole 0.003 mm in diameter.

When 5 minutes has elapsed after the piercing, the container will contain …

88%	1.	more A than B
6%	2.	equal amounts of each gas
0%	3.	less A than B
6%	4.	Further information needed

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