Half-life calculations

Notes on General Chemistry

http://quantum.bu.edu/notes/GeneralChemistry/HalfLifeCalculations.pdf
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Using coin flips to count people

> Everyone in of a group of 202 people is initially standing. Each person flips a coin, and sits down if the coin lands heads up. Each person still standing flips again, and sits down if the coin lands heads up. After 5 coin flips, 7 people remain standing. How many people are in the large group?

The fundamental expression

$$N_n = (1/2)^n N_0.$$

becomes

$$7 = (1/2)^5 N_0.$$

We rearrange this for the number of people present, then the initial number of students—the class attendance—evaluates to

$$N_0 = 7 \times 2^5 = 7 \times 64 = 448.$$

This is different from the actual number of people, 202, but the result shows that, to within a factor of two, we can use "half life decay" to "count" an initial population size.

> Why is our result so different form the number of people present?

For such a small sample, this is definitely the hard and not very accurate way to count. But if we had a very large sample, say the people at a Boston Red Sox game in Fenway Park, it would be much quicker than counting each person individually.

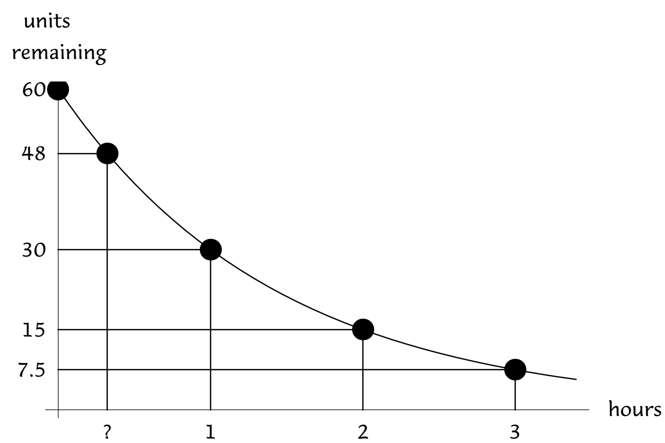
Half-life

Half-life, t_{half}, is defined as the amount of time required for the amount of a substance to be reduced by 50%. Half-life a useful concept if its value does *not* depend on how much material is present. Such a decay process is called d *first order*. Nuclear decay is first order.

Let's say a substance undergoes first order decay with $t_{\text{half}} = 1$ hour, and that we begin with 60 units of the substance. This means

t/hours	units remaining
0	60
1	30 = 50% of 60
2	15 = 50% of 30
3	7.5 = 50% of 15

Here is a plot of the amount remaining versus time.



Decay of 60 units with t_{half} = 1 hour. What is the time, marked ?, at which 48 units remain?

The curve connecting the points is the known as the first-order decay curve. We can derive an expression for this curve, and so the amount remaining at any time (not just a multiple of t_{half}). First, we rearrange the fundamental relation

$$N_n = (1/2)^n N_0.$$

and then take the natural logarithm of both sides to get

$$\ln(N_n / N_0) = \ln[(1/2)^n] = -n \ln(2),$$

since $\ln(1/2) = \ln(1) - \ln(2) = -\ln(2)$. Then, by taking the antilogarithm of both sides,

$$N_n / N_0 = e^{-n \ln(2)},$$

we can express the population after $n = t / t_{half}$ half lives as

$$N_n = N_0 \, e^{-n \ln(2)} = N_0 \, e^{-t \ln(2)/t_{half}} = N_0 \, e^{-0.69 \, t/t_{half}}.$$

For numerical calculations without calculator, it is more convenient to work with base-10 logarithms. Using the same procedure as above, you will get the expression

$$N_n = N_0 \, 10^{-n \log(2)} = N_0 \, 10^{-0.30 \, t/t_{half}}.$$

> Show that this expression is correct.

Given half-life, how long to decay by a *x*%?

Let's determine the missing time on the graph above.

> Given 60 units of a substance that decays with t_{half} = 1 hour, how much time must elapse for 48 units to remain?

We can use the fundamental expression

$$N_n = (1/2)^n N_0.$$

to write

$$48 = (1/2)^n\ 60,$$

and then solve this for the number of half-lives that have elapsed. Taking logs of both sides and rearranging, we get

$$-n \log(2) = \log(48/60) = \log(0.80) = \log(2^3 \times 10^{-1}) = -1 + 3 \log(2).$$

This means

$$n \log(2) = \frac{1 - 3 \log(2)}{\log(2)} = \frac{1 - 0.90}{0.30} = \frac{0.10}{0.30} = 1/3.$$

Since $t_{\text{half}} = 1$ hour, in $1/3$ hour = 20 minutes, 12 units will have decayed.

Given a decay amount in a given time, what is half-life?

If we know the amount of decay that has taken placed in a given time, we can determine the half-life.

> A sample decays by 90.% in 20. min. What is its half-life?

As always, let's begin with the fundamental expression

$$N_n = (1/2)^n N_0.$$

In this case we know that in 20. min,

$$(1/2)^n = \frac{N_n}{N_0} = \frac{0.10\ N_0}{N_0} = 0.10.$$

Solving for n we get

$$-n \log(2) = \log(0.10.) = \log(1.0 \times 10^{-1}) = -1,$$

$$n = t/t_{\text{half}} = 1/\log(2) = 1/0.30 = 20\ \text{min}/t_{\text{half}},$$

and so that $t_{\text{half}} = 0.30 \times 20.\ \text{min} = 6.0\ \text{min}$.