Blind chance & dumb luck

Everything—absolutely everything—that happens, happens solely because of blind chance and dumb luck.

To quantify this...
1. Learn to count the ways
2. Search for greatest number of ways

Counting distinguishable (unique) arrangements

Say we have three girls and four boys. What is the probability of calling them into line in the order gggbbbb?

\[
\frac{1}{7} \times \frac{2}{6} \times \frac{1}{5} = \frac{1}{35}
\]
Counting *distinguishable* (unique) arrange

Say we have three girls and four boys.

What is the probability of calling them into line in the order $bgbgbg$?

$$\frac{\frac{2}{7} \times \frac{2}{6} \times \frac{2}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{1}{2}}{3} = \frac{1}{35}$$

Counting *distinguishable* (unique) arrange

Say we have three girls and four boys.

If we ignore whether a child is boy or girl, what is the total number of arrangements?

$$7 \times 6 \times 5 \times \cdots \times 1 = 7! = 5040$$

Counting *distinguishable* (unique) arrange

Say we have three girls and four boys.

For each particular arrangement, say $bgbgbg$, how many ways can it come about?

$$3! \times 4! = 144$$

Counting *distinguishable* (unique) arrange

Say we have three girls and four boys.

This means the total number of arrangements can be expressed as

$$7! = W \times 3! \times 4!$$

$$W = \frac{5040}{144} = 35$$
Counting *distinguishable* (unique) arrange

More generally, say we have $j$ girls and $k$ boys.

The number of unique arrangements of $n_1$ objects of one kind and $n_2$ objects of another kind is

$$W(j, k) = \frac{j!k!}{j+k!}$$

Practice with particle dispersal


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**[TP]** How many *distinguishable* ways can 3 water molecules and 2 ink molecules be arranged?

- 0% 1. 8
- 0% 2. 10
- 0% 3. 12
- 0% 4. 120
- 0% 5. None of these

**[TP]** How many *distinguishable* ways can 5 water molecules and 2 ink molecules be arranged?

- 0% 1. 14
- 0% 2. 21
- 0% 3. 240
- 0% 4. 5040
- 0% 5. None of these
[TF] How many distinguishable ways can 2 ink molecules be arranged among 12 water molecules?

- 1. 36
- 2. 240
- 3. 455
- 4. 720
- 5. None of these

Maximum particle dispersal = uniform pressure

Pressure in a gas is unequal
Pressure in a gas is **uniform**

Permeable barrier

Pressure in a gas **becomes uniform**

Why?

Lattice gas model of pressure

\[
\frac{1}{RT} P = \frac{n}{V} = \text{gas density} \\
n = \text{particles} \\
V = \text{lattice positions}
\]

\[
P_{\text{left}} > P_{\text{right}}
\]

\[
\begin{align*}
\text{Left side: } n/V &= 2/4, \quad W_{\text{left}} = \ldots \\
&= 6 \\
\text{Right side: } n/V &= 1/8, \quad W_{\text{right}} = \ldots \\
&= 8 \\
W_{\text{total}} &= W_{\text{left}} \times W_{\text{right}} = 6 \times 8 = 48
\end{align*}
\]
Pressure in a gas becomes uniform

Why?

- $P_{\text{left}} > P_{\text{right}}$ has $W_{\text{total}} = 48$
- $P_{\text{left}} = P_{\text{right}}$ has $W_{\text{total}} = 112$
- $P_{\text{left}} < P_{\text{right}}$ has $W_{\text{total}} = 56$

Uniform pressure maximizes $W$!
\[ S = k_B \ln(W) \]

Why natural log?

Doubling size of system: \( W \rightarrow W \times W = W^2 \)

Doubling size of system: \( S \rightarrow 2S \), so ...

Boltzmann’s definition makes \( S \) scale with size of system (extensive).

\[ k_B = \frac{R}{N_A} = 1.4 \times 10^{-23} \text{ J/K} \]

Spontaneous?

Calculate the entropy change.

\[ W_f = 1 \rightarrow W_f = (6 + 3)!/(6! 3!) = 84 \]

\[ \Delta S = S_f - S_i = k_B \ln(W_f/W_i) = k_B \ln(84/1) > 0 \]